4. H. C. Van de Hulst, *Light Scattering by Small Particles.* Wiley, New York (1957).
5. W. J. Wiscombe, The delta-M method. Rapid yet accurate radiative flux calculations for strongly asymmetric phase functions, *J. Atmos. Sci.* **34,** 1408–1422 (1977).
6. B. Davison, *Neutron Transport Theory.* Oxford University Press, London (1958).
7. G. W. Kattawar and G. N. Plass, Electromagnetic scattering from absorbing spheres, *Appl. Optics* **6,** 1377–1382 (1967).
8. T. J. Love and R. J. Grosh, Radiative heat transfer in absorbing, emitting and scattering media, *J. Heat Transfer* **C87,** 161–166 (1965).
9. W. J. Wiscombe and G. W. Grams, The backscatter fraction in two-stream approximations, *J. Atmos. Sci.* **33,** 2440–2451 (1976).

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MULTI-PRANDTL NUMBER CORRELATION EQUATIONS FOR NATURAL CONVECTION IN LAYERS AND ENCLOSURES

K. G. T. Hollands

Department of Mechanical Engineering, University of Waterloo, Waterloo N2L 3G1, Canada

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INTRODUCTION

An earlier note [1] reported an expression which closely fits the experimental *Nu–Ra* data relevant to natural convection in horizontal heated-from-below layers of a fluid (water) having a Prandtl number of about 6. The present note has two purposes: (1) to point out that a slightly altered form of that expression closely fits all the reliable available experimental data relevant to horizontal layers, regardless of the Prandtl numbers; and (2) to point out that a slightly altered form of this new expression fits the available experimental data relevant to horizontal *enclosures* as well as layers. (For the purposes of this note a horizontal enclosure is one bounded only by horizontal and vertical surfaces, and a horizontal layer is a horizontal enclosure whose horizontal dimensions have been made so large with respect to the vertical ones that they have ceased to effect the Nusselt number. See the earlier note [1] for definitions of *Nu* and *Ra*.)

HORIZONTAL LAYERS

The earlier note [1] gave *Nu–Ra* expressions for both air ($Pr \approx 0.7$) and water ($Pr \approx 6$) and demonstrated a definite Prandtl number dependence inside the range $0.7 \lesssim Pr \lesssim 6$. That the Prandtl number dependence persists outside that range is demonstrated in Fig. 1, which shows the data of Schmidt and Silveston [2] at $Pr = 35$, 100, and 3000, Rossby [3] at $Pr = 200$ and 0.025, and Globe and Dropkin [4] at various *Pr*, together with plots of the earlier note's expressions for air and water. The consistency of the Schmidt and Silveston data with the Rossby data (or vice versa) is both striking and reassuring. But the Globe and Dropkin data is generally inconsistent with the other data. A study of the Globe and Dropkin experiment reveals sources for experimental error large enough to explain the inconsistency;* hence their data will be ignored in what follows.

The proposed altered form of the previous note's expression is

$$Nu = 1+[1-1708/Ra]^{\bullet}[k_1+2(Ra^{1/3}/k_2)(1-\ln(Ra^{1/3}/k_2))] + [(Ra/5830)^{1/3}-1]^{\bullet}, \quad (1)$$

* For example the hot plate was not guarded on either the bottom or the sides, and no special precautions were taken to ensure the isothermality of each plate.

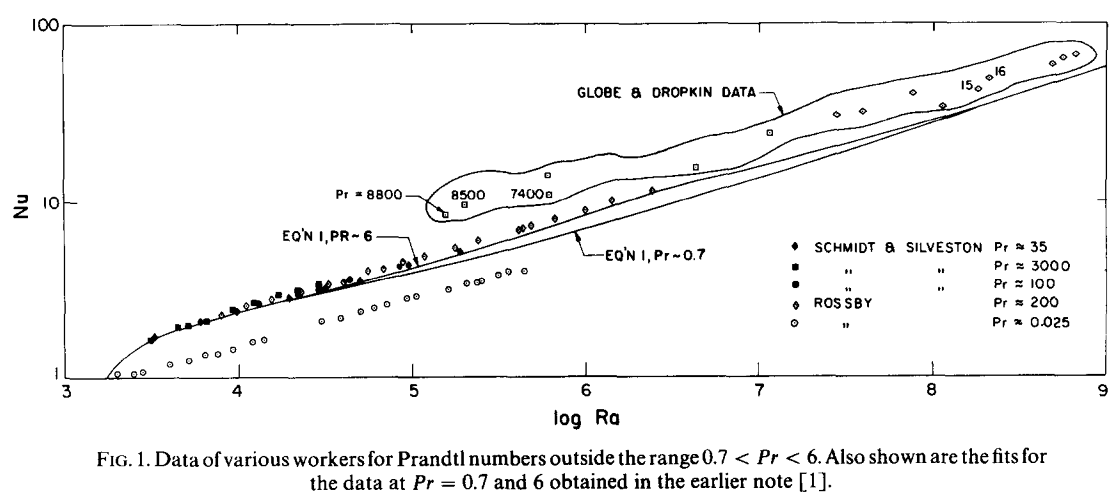

Fig. 1. Data of various workers for Prandtl numbers outside the range $0.7 < Pr < 6$. Also shown are the fits for the data at $Pr = 0.7$ and 6 obtained in the earlier note [1].

Table 1. Values of k_1 and k_2 to be used in equation (1)

Pr (approx.)	k_1	k_2	Range of Ra tested	Reference
0.024	0.35	> 200	$10^3 \leqslant Ra \leqslant 5\times10^5$	[3]
0.7	1.40	> 400	$10^3 \leqslant Ra \leqslant 10^8$	[1]
6	1.44	140	$10^3 \leqslant Ra \leqslant 10^{11}$	[1]
34	1.44	100	$10^3 \leqslant Ra \leqslant 2\times10^5$	[2]
100	1.44	~ 85	$10^3 \leqslant Ra \leqslant 10^5$	[2]
200	1.44	85	$10^3 \leqslant Ra \leqslant 3\times10^6$	[3]
3000	1.44	~ 75	$10^3 \leqslant Ra \leqslant 5\times10^4$	[2]

where, as in ref. [1], square brackets with a dot: $[\ ]^{\bullet}$ indicate that only positive values of the argument inside the brackets are to be taken (if the argument is negative, the quantity is to be taken as equal to zero). The parameters k_1 and k_2 are functions of the Prandtl number; the values which they must take in order to make the equation fit the data are given in Table 1. The following equations fit the dependences of k_1 and k_2 exhibited by Table 1

$$k_1 = 1.44/(1+0.018/Pr+0.00136/Pr^2), \quad (2)$$

$$k_2 = 75\exp(1.5Pr^{-1/2}). \quad (3)$$

Gough *et al.* [5] suggested the form of equation (2); equation (3) is empirical.

For all practical purposes, equations (1)–(3) with $Pr = 0.7$ or 6 reduce to the equations given in the earlier note [1], so they agree closely with air and water data. Figure 2 shows the comparison with the data at other Prandtl numbers. The agreement is excellent, generally within a few per cent. Further experiments are required to establish the full range of validity.

HORIZONTAL ENCLOSURES

Every fluid-filled horizontal enclosure is characterized by a critical Rayleigh number Ra_c, which depends upon the enclosure's various geometric and thermal properties. As explained by Catton [6], early theoretical methods of obtaining Ra_c relied on an approximate 'adjusted wave number' technique, but since 1967 exact solutions for Ra_c have become available for the principal cross-sectional shapes of interest [7–11], and the methodology for finding Ra_c for other shapes is now straightforward. Although the available exact solutions are for opaque fluids, the methods of Edwards and Sun [12, 13] and Sun [14] may be used to approximate the alteration required to account for the radiative effects. A summary of the results of the exact-theory approach has been recently prepared [15]. Methods of predicting Ra_c are core to any method of correlating the experimental Nu–Ra data.

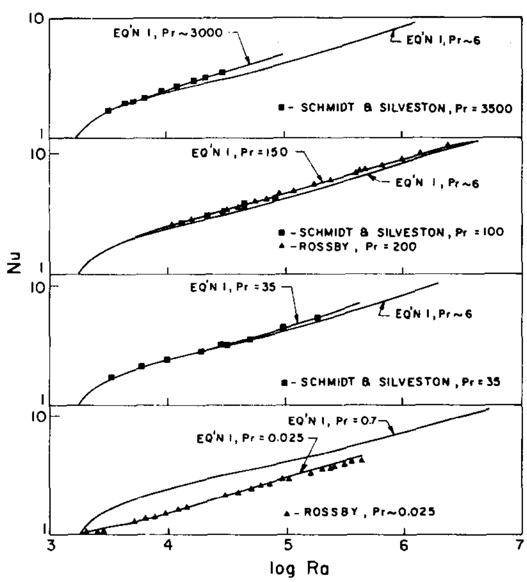

FIG. 2. Comparison of data at $Pr = 3000$, 150, 35, and 0.025 with equation (1).

The available experimental Nu–Ra data relevant to horizontal enclosures [16–20] have been correlated with some success using a combined technique which incorporates both the adjusted wave number method and the power integral method [16, 17, 21]. Despite its success this combined technique has three shortcomings. First it predicts the Nu–Ra relation to be independent of Prandtl number—a fact at variance with the experimental evidence [18]. Second it fails to predict the 1/3 power law dependence of Nu on Ra at high Rayleigh number.* Third it is knitted into the older adjusted wave number method of finding Ra_c. In light of these shortcomings, new methods of correlating Nu–Ra data, such as that described below, are of interest.

Two modifications were necessary to make equation (1) fit the enclosure data: replacing the 1708 in the second term by the Ra_c appropriate to the enclosure in question, and multiplying the third term by a factor such that that term contributes only at $Ra > Ra_c$, while maintaining the correct high Ra asymptote. An exponential factor, similar to one used previously [18], was adopted. The modified equation is

$$Nu = 1+[1+x_1^{-3}]^{\bullet}[k_1+2x_2^{(1-\ln x_2)}]$$
$$+[x_3-1]^{\bullet}[1-\exp(-0.95[x_1-1]^{\bullet})], \quad (4)$$

where $x_i = (Ra/Ra_i)^{1/3}$, with $Ra_1 = Ra_c$; $Ra_2 = k_2^3$; and $Ra_3 = 5930$. When $Ra_c = 1708$, equation (4) yields Nusselt numbers so close to those from equation (1) that for practical purposes one can say that equation (4) reduces to equation (1) as a special case. Figure 3 shows a plot of equation (4) for $Pr = 6$ with Ra_c as parameter.

In order to compare equation (4) with the relevant enclosure data the measured values of Ra_c were used in the equation instead of the theoretical values. The inset of Fig. 3 shows the resulting comparison for several sets of data. In the experiments of Sun and Edwards and Catton and Edwards, the Prandtl number varied, and some of the deviations of their data from the smooth curve of equation (4), which is based on their median Prandtl number of 600, may be explained as being due to this variation in Prandtl number. To illustrate, the dashed lines show the range in Nu that would be expected from equation (4) due to the Prandtl number range (from 120 to 1700) covered by the experiments of Catton and Edwards on conducting walls.

Unfortunately, not all of the sets of literature data agree as closely with equation (4) as the selected data

* Goldstein and Tokuda [22] recently validated the 1/3 law asymptote by measurements on water in the $10^{10} < Ra < 10^{11}$ range, obtaining $Ra = 0.0556Ra^{1/3}$. (This is precisely the asymptote predicted by equation (1): at high Ra the third term dominates, so $Nu \rightarrow (5800^{-1/3})Ra^{1/3} = 0.0556Ra^{1/3}$.)

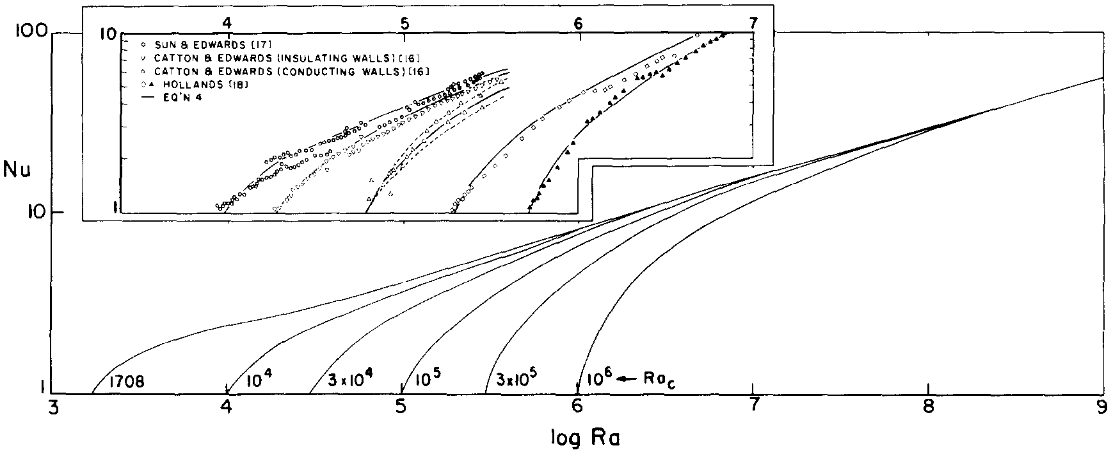

FIG. 3. A plot of equation (4) for various Ra_c when $Pr = 6$, and (inset) a comparison of selected data with equation (4).

of Fig. 3, although most of it does. Certain of the data, principally the data of Smart *et al.* for long rectangle cross-sections, showed maximum deviations from equation (4) of up to 25%. No particular cell characteristic, other than a long rectangular cross-section, seems to control the presence of these larger deviations.

The chief advantage of equation (4) is that it reduces all the complexities caused by side wall conductive and radiative interactions to finding the appropriate Ra_c. Moreover, if $Ra_c = 1708$, it reduces to equation (1). It is therefore suitable for both enclosures and layers.

REFERENCES

1. K. G. T. Hollands, G. D. Raithby and L. Konicek, Correlation equations for free convection heat transfer in horizontal layers of air and water, *Int. J. Heat Mass Transfer* **18,** 879–884 (1975).
2. E. Schmidt and P. L. Silveston, Natural convection in horizontal liquid layers, *Chem. Engng Prog. Symp. Ser.* **55**(29), 163–169 (1959).
3. H. T. Rossby, A study of Bénard convection with and without rotation, *J. Fluid Mech.* **36**(2), 309–335 (1969).
4. S. Globe and D. Dropkin, Natural convection heat transfer in liquids confined by two plates and heated from below, *J. Heat Transfer* **81,** 24–28 (1959).
5. D. O. Gough, E. A. Spregel and J. Toomre, Modal equations for cellular convection, *J. Fluid Mech.* **68**(4), 695–719 (1975).
6. I. Catton, Natural convection in enclosures, *Proc. 6th Int. Heat Transfer Conf.*, Toronto, Vol. 6, pp. 13–31 (1978).
7. S. H. Davis, Convection in a box: linear theory, Rand Report No. RM-5251-PR, The Rand Corp., June (1967).
8. I. Catton, Convection in a closed rectangular region: the onset of motion, *J. Heat Transfer* **92,** 186–187 (1970).
9. I. Catton, The effect of insulating vertical walls on the onset of motion in a fluid heated from below, *Int. J. Heat Mass Transfer* **15,** 665–672 (1972).
10. I. Catton, Effect of wall conduction on the stability of a fluid in a rectangular region heated from below, *J. Heat Transfer* **94**(4), 446–452 (1972).
11. J. C. Buell, The effect of rotation and wall conductances on the stability of a fully enclosed fluid heated from below, M.Sc.E. thesis, University of California, Los Angeles, School of Engineering and Applied Science, Los Angeles, California (1981).
12. D. K. Edwards and W. M. Sun, Effect of wall radiation on thermal instability in a vertical cylinder, *Int. J. Heat Mass Transfer* **14,** 15–18 (1971).
13. D. K. Edwards and W. M. Sun, Prediction of the onset of natural convection in rectangular honeycomb structures, Int. Solar Energy Soc. Conf., Melbourne, Australia, Paper No. 7/62 (1970).
14. W. M. Sun, Effect of arbitrary wall conduction and radiation on free convection in a cylinder, Ph.D. thesis, University of California, Los Angeles, California (1970).
15. G. D. Raithby and K. G. T. Hollands, Natural convection, in *Handbook of Heat Transfer Fundamentals*, Chap. 6. McGraw-Hill, New York (1984) in press.
16. I. Catton and D. K. Edwards, Effect of side walls on natural convection between horizontal plates heated from below, *J. Heat Transfer* **89,** 295–299 (1967).
17. W. M. Sun and D. K. Edwards, Natural convection in cells with finite conducting side walls heated from below, *Proc. 4th Int. Heat Transfer Conf.*, Versailles, France, Paper No. NC 2.3 (1970).
18. K. G. T. Hollands, Natural convection in horizontal thin-walled honeycomb panels, *J. Heat Transfer* **95**(4), 439–444 (1973).
19. R. L. D. Cane, K. G. T. Hollands, G. D. Raithby and T. E. Unny, Free convection heat transfer across inclined honeycomb panels, *J. Heat Transfer* **99**(1), 86–91 (1977).
20. D. R. Smart, K. G. T. Hollands and G. D. Raithby, Free convection heat transfer across rectangular celled diathermous honeycombs, *J. Heat Transfer* **102**(1), 75–80 (1980).
21. D. K. Edwards and I. Catton, Prediction of heat transfer by natural convection in closed cylinders heated from below, *Int. J. Heat Mass Transfer* **12,** 23–30 (1969).
22. R. J. Goldstein and S. Tokuda, Heat transfer by thermal convection at high Rayleigh numbers, *Int. J. Heat Mass Transfer* **23,** 738–740 (1980).